Swiss Federal Institute of Technology

Computer Engineering and Networks Laboratory

Cyclic Dependencies in Modular Performance Analysis

TEC Group meeting

Computer Engineering and Networks Laboratory

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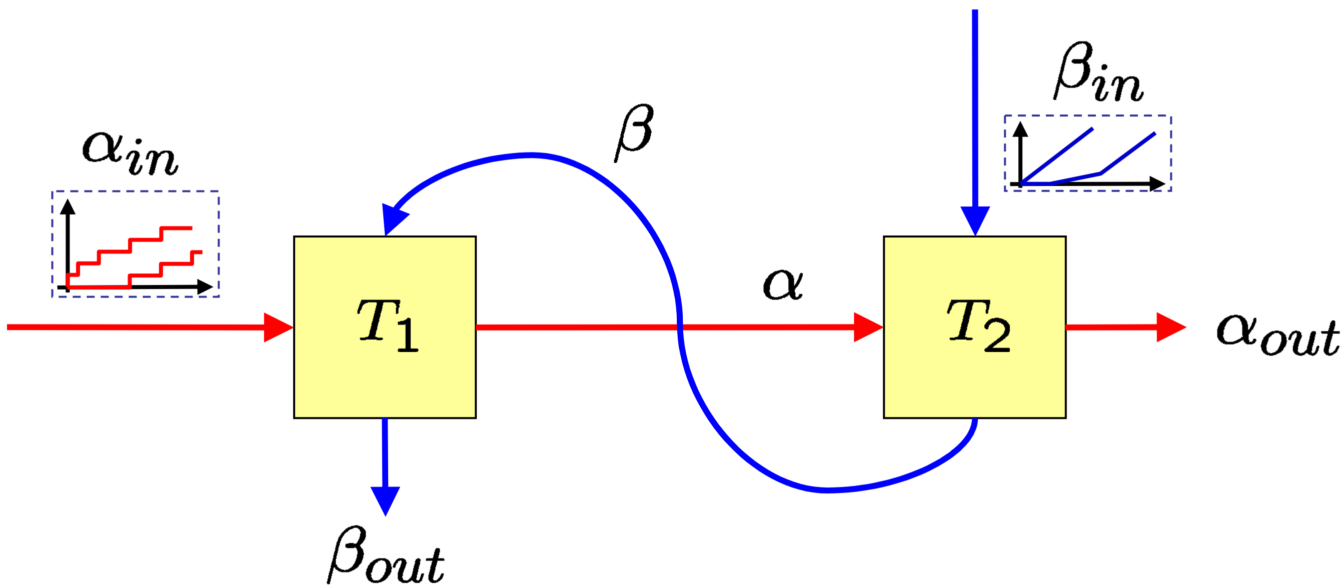
Motivation

- MPA-RTC used successfully on acyclic systems
- For systems with cyclic dependencies the foundations of the formalism are less well understood

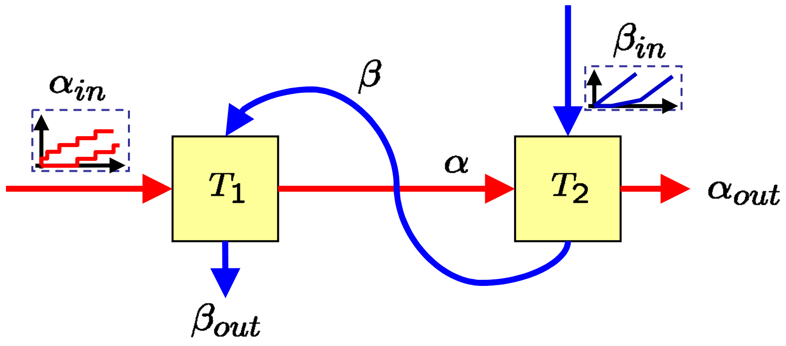


Fixed point iteration

$$\Sigma_{in} := (\alpha^l_{in}, \alpha^u_{in}, \beta^l_{in}, \beta^u_{in})$$

$$\Sigma_h := (\alpha^l, \alpha^u, \beta^l, \beta^u)$$

$$\Sigma := (\Sigma_{in}, \Sigma_h)$$



Ψ : mapping $\Sigma \rightarrow \Sigma'$ according to transfer functions of RTC

Fixed point Σ^*: solution of equation $\Sigma = \Psi(\Sigma)$

Starting from an initial approximation Σ^0 we can compute the sequence $\Sigma^0, \Sigma^1, \Sigma^2, \ldots$ with $\Sigma^{k+1} = \Psi(\Sigma^k)$ hoping that it converges to a limit Σ^*

Questions

- Will any fixed point of Ψ correctly characterize all possible behaviors of the system ?
- Can there be several fixed points ?
- If so, is there an optimal fixed point
(i.e. one that provides tighter bounds than all others) ?
- Can an (optimal) fixed point be computed as the limit of a sequence $\Sigma^0, \Sigma^1, \Sigma^2, \ldots$ of approximations ?
- Will the iteration always converge to a limit Σ^* ?
- How to choose the initial approximation Σ^0 ?

Related work

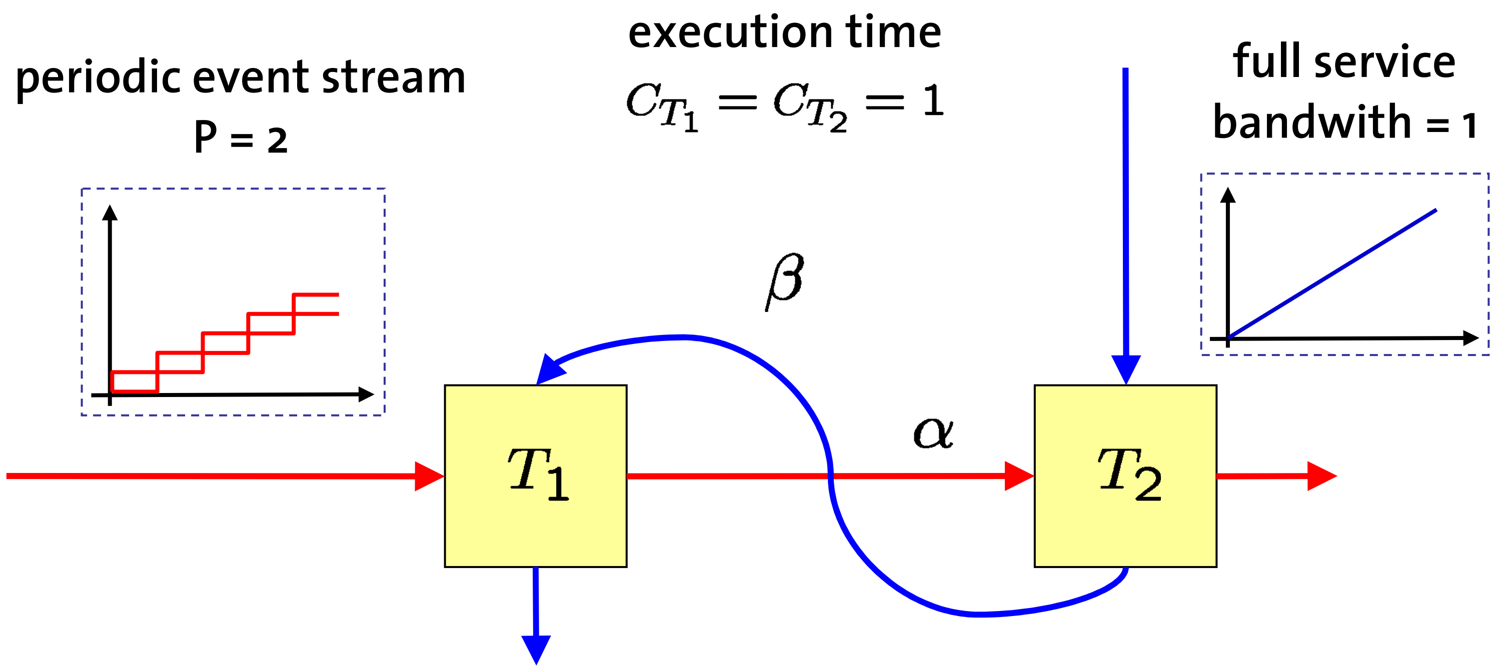
- **[Jersak, Richter, Ernst - 2005]**
  - Consider cyclic dependencies in periodic-with-jitter event model
  - Only informal statements about convergence properties

- **[Schiøler et al. - 2005]**
  - First results on Cyclic Network Calculus
  - Several implicit assumptions
  - Ignore problem of zero-delay cycles

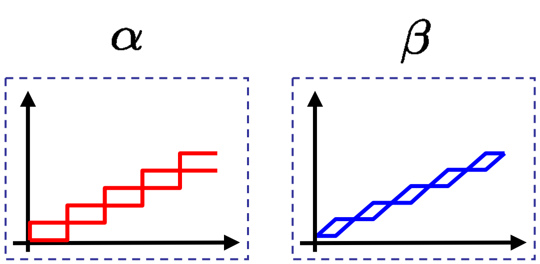
Contributions

- We introduce a simple operational model of distributed systems and develop a general operational semantics underlying the Real-Time Calculus
- On this basis we show that the behavior of cyclic systems can be analyzed by fixed point iteration
- We prove central properties about faithfulness of fixed points computed with RTC
- We provide a method that leads to the optimal fixpoint

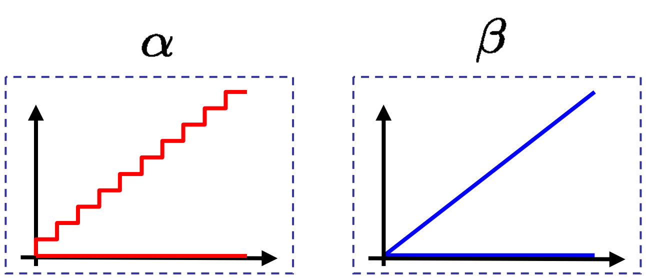
In general fixed points are not unique...



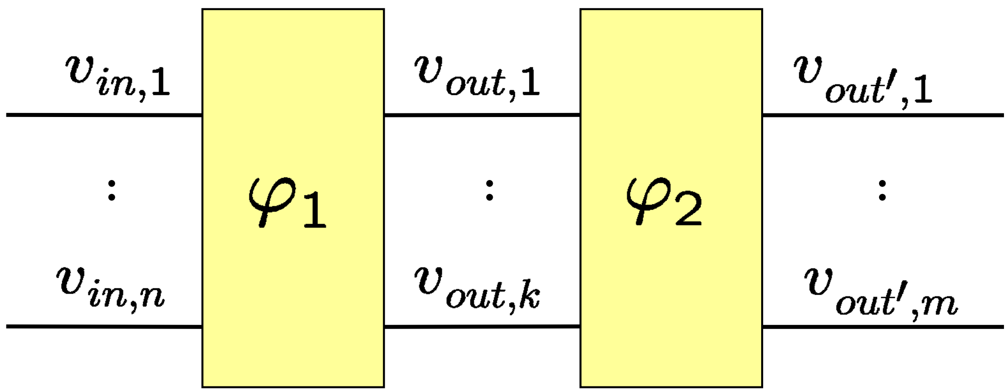
Fixed point 1 (optimal):



Fixed point 2 (wide):



Operational model (1)



V : set of streams

$Tr(V)$: set of traces on streams V

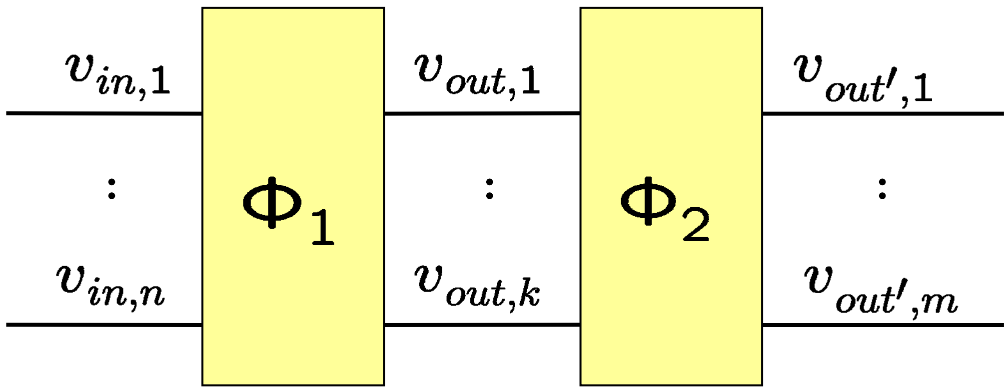
Trace: $\sigma : V \mapsto ((\mathbb{R} \times \mathbb{R}) \mapsto \mathbb{R}^{\geq 0})$

e.g. $\sigma(v)(s,t)$:= number of events in interval $[s,t)$ on stream v

Component = trace mapping $\varphi : Tr(V_{in}) \mapsto Tr(V_{out})$

System = trace transformer $\psi : Tr(V_{in} \cup V_{out}) \mapsto Tr(V_{in} \cup V_{out})$

Operational model (2)



V : set of streams

$Char(V)$: set of characterizations on streams V

"Characterization = set of traces"

Stream characterization: $\Sigma : V \mapsto 2^{((\mathbb{R}\times\mathbb{R})\mapsto\mathbb{R}^{\geq 0})}$

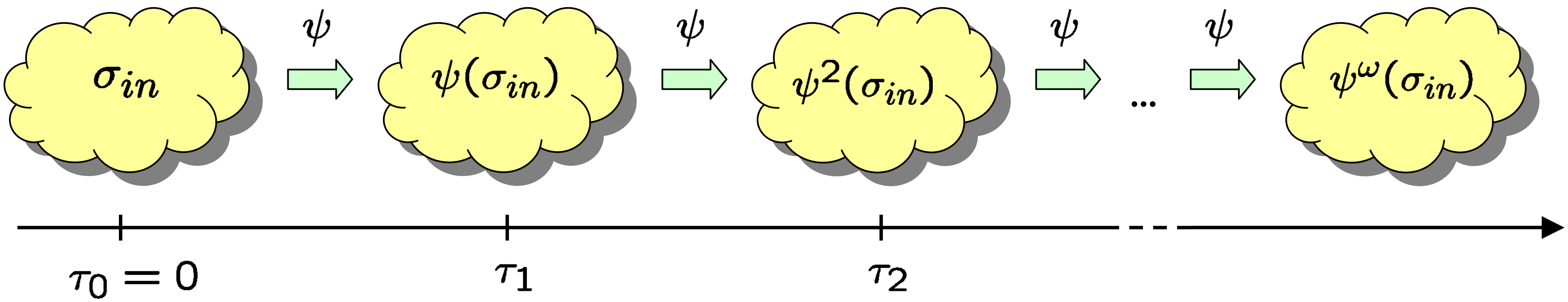
e.g. $\Sigma(v)(s,t)$:= bounds on number of events in interval $[s,t)$ on stream v

$$\sigma \models \Sigma \quad \text{iff} \quad \forall v \in V \; \sigma(v) \in \Sigma(v)$$

Component = characterization mapping $\Phi : Char(V_{in}) \mapsto Char(V_{out})$

System = characterization transformer $\Psi : Char(V_{in} \cup V_{out}) \mapsto Char(V_{in} \cup V_{out})$

Assumption: Simulatable system (1)



$$\forall i \geq 0 \qquad \sigma \simeq_{\tau_i} \sigma' \quad \Rightarrow \quad \psi(\sigma) \simeq_{\tau_{i+1}} \psi(\sigma')$$

"Any trace on V_{in} induces a unique trace on V_{out}"

Assumption: Simulatable system (2)

This assumption excludes:

- Non-deterministic components
- Zero-delay cycles

In MPA-RTC a system is simulatable according to the previous definition if:

- The time needed to process events on a components is > 0
- The system does not contain any cycle of resource streams

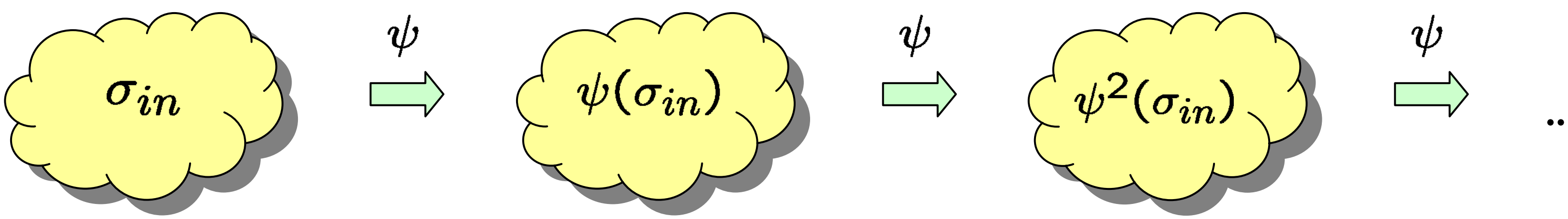
① Correctness of fixed point

Question: Given Ψ, Σ_{in}, $\sigma_{in} \models \Sigma_{in}$

Will the trace $\psi^{\omega}(\sigma_{in})$ on input σ_{in} satisfy the limit of a sequence $\Sigma^0, \Psi(\Sigma^0), \Psi(\Psi(\Sigma^0)), \ldots$?

If Σ^0 is satisfiable and also agrees with Σ_{in} on V_{in} then

$$\psi^{\omega}(\sigma_{in}) \models_{\leq \tau_i} \Psi^i(\Sigma^0) \quad \forall i \geq 0$$



$\models_{\leq 0} \Sigma^0$ $\quad$ $\models_{\leq \tau_1} \Psi(\Sigma^0)$ $\quad$ $\models_{\leq \tau_1} \Psi^2(\Sigma^0)$

Correctness of fixed point

$\Rightarrow$

- The iteration is safe
- If it converges then the fixed point is correct

② Convergence

Question: When will the sequence $\Sigma^0, \Psi(\Sigma^0), \Psi(\Psi(\Sigma^0)), \ldots$ converge ?

$\models$ introduces a partial order $\sqsubseteq$ on $Char(V)$:

$\Sigma \sqsubseteq \Sigma'$ iff $\sigma \models \Sigma \Rightarrow \sigma \models \Sigma' \quad \forall \sigma \in Tr(V)$

Assume $(Char(V), \sqsubseteq)$ constitutes a chain-complete poset and Ψ is monotone and continuous

$\Longrightarrow$ Ψ has unique smallest fixpoint Σ^*

$\Sigma^0, \Psi(\Sigma^0), \Psi(\Psi(\Sigma^0)), \ldots \rightarrow \Sigma^*$ if

- Σ^0 agrees with Σ_{in} on V_{in}
- Σ^0 is satisfied by at least one actual system trace
- $\Sigma^0 \sqsubseteq \Sigma^*$

③ Initial approximation

Question: How to choose Σ^0 ?

1. Construct σ such that $\sigma|_{V_{in}} \models \Sigma_{in}$ and satisfies Σ^*

e.g. Find one actual system trace by simulation

2. Let Σ_σ be the tightest characterization for σ

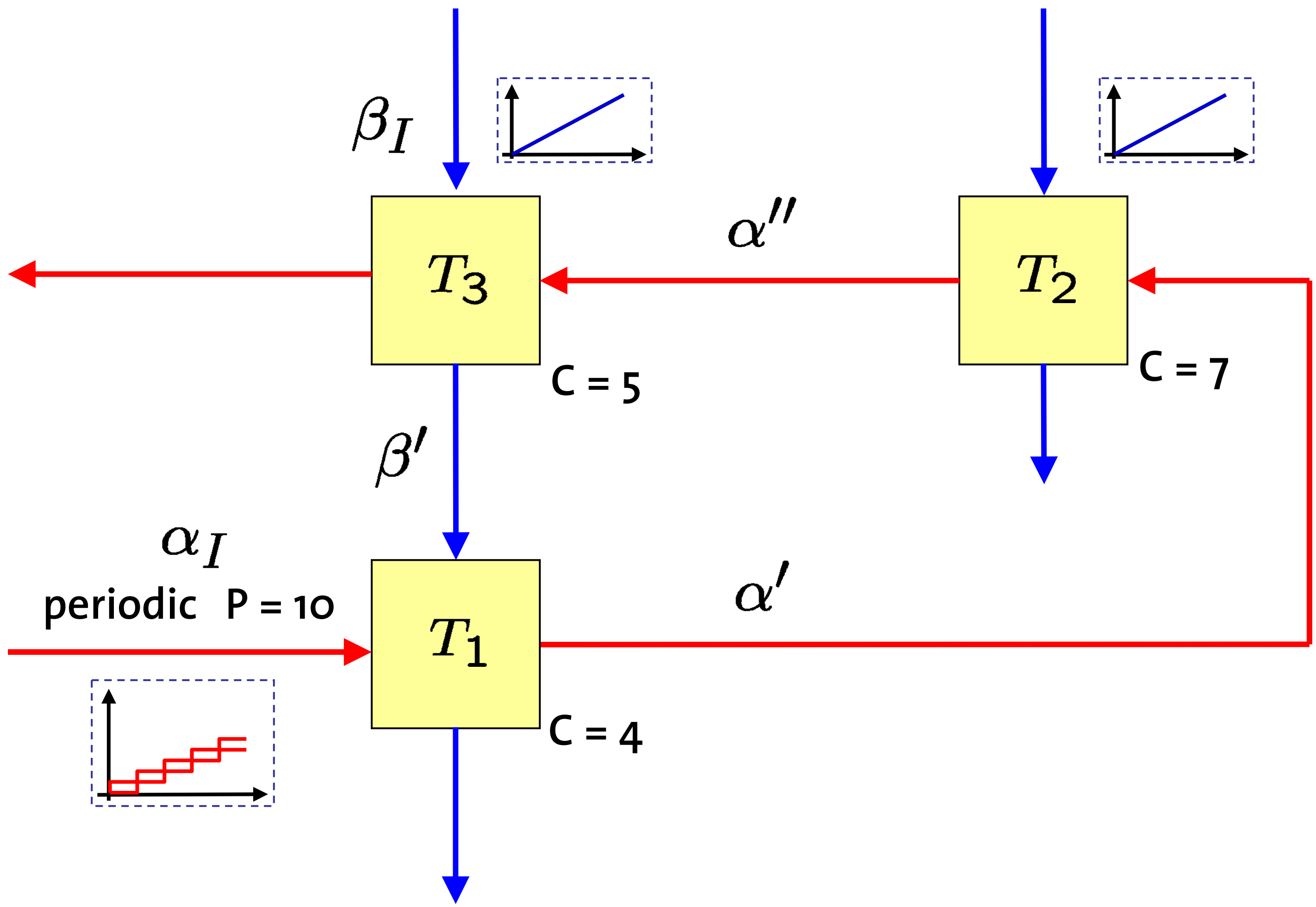
Use Σ_σ as initial approximation Σ_0

Alternative:
long term rates
[Schiøler 2005]

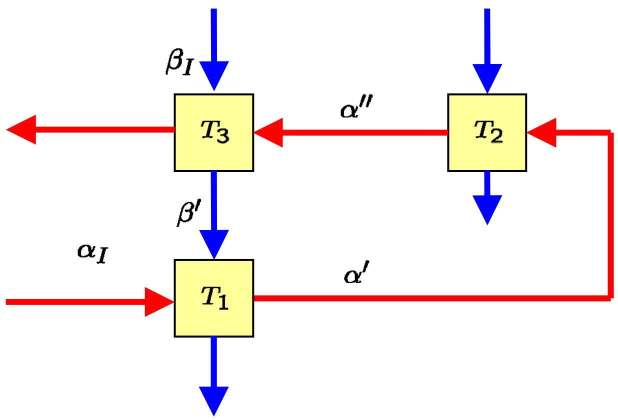
3) Perform fixed point iteration

Guaranteed convergence to Σ^* (optimal fixed point of Ψ)

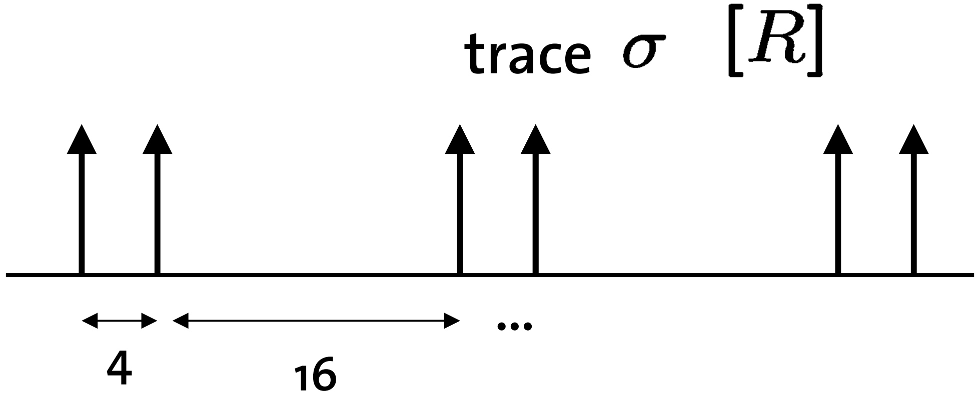
Example

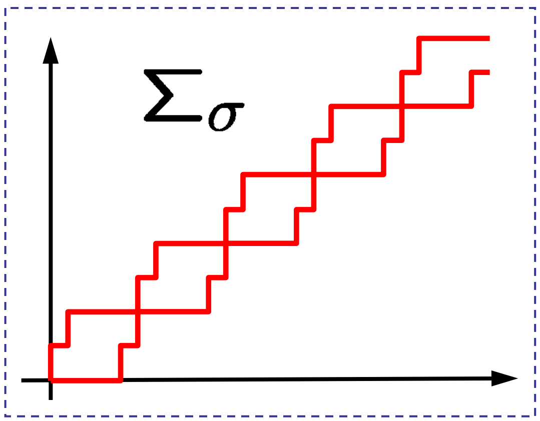


Example (1)



Simulation:

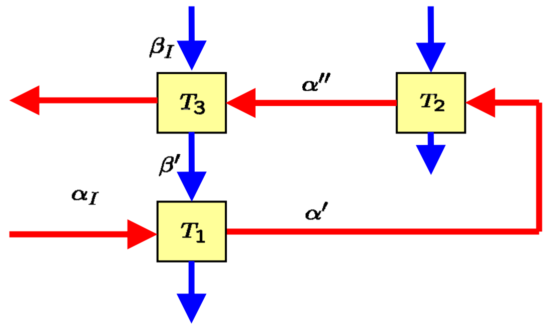




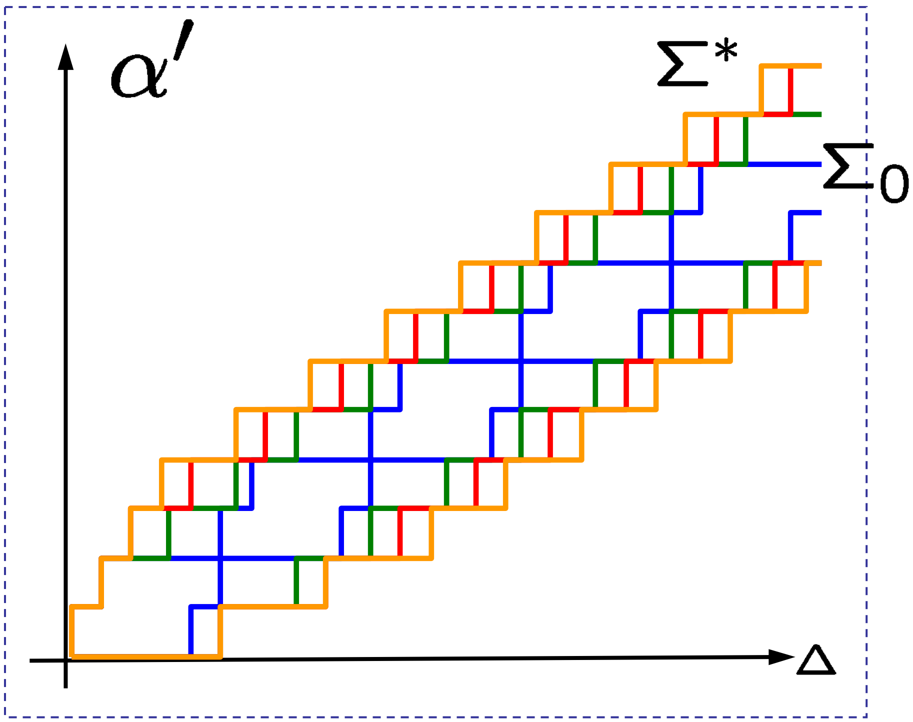
$$\alpha'^{l} = R \overline{\oslash} R$$

$$\alpha'^{u} = R \oslash R$$

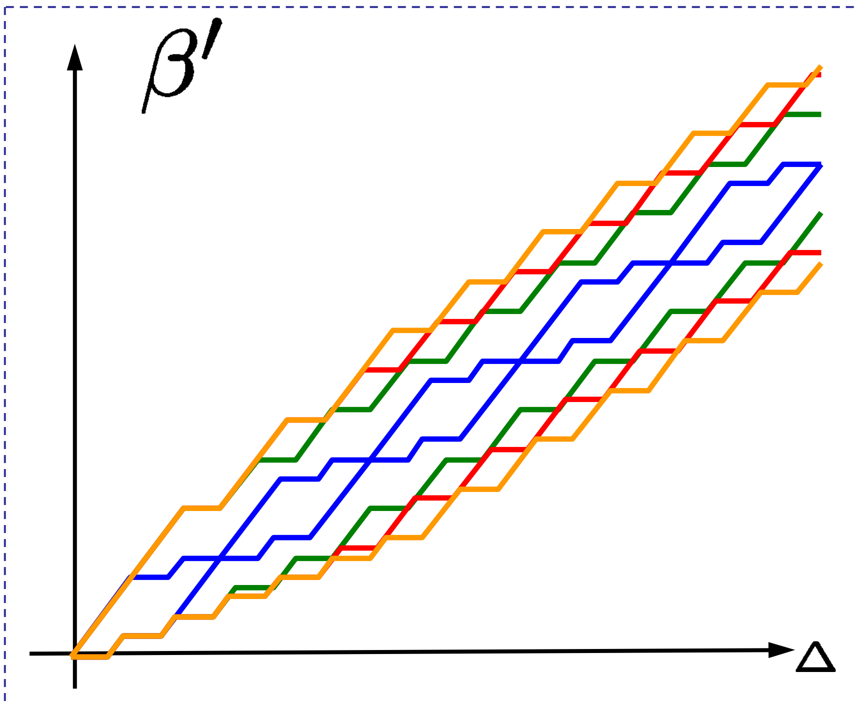
Example (2)



β_I
T_3
α''
T_2
β'
α_I
T_1
α'

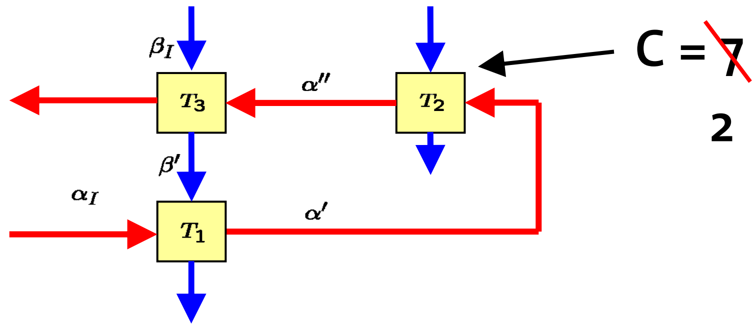


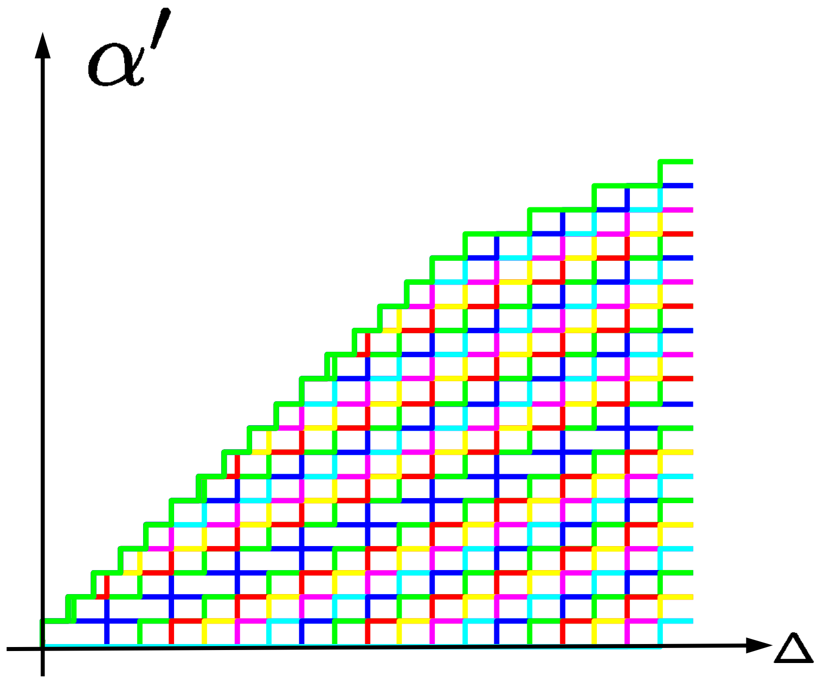
α'
Σ^*
Σ_0
Δ

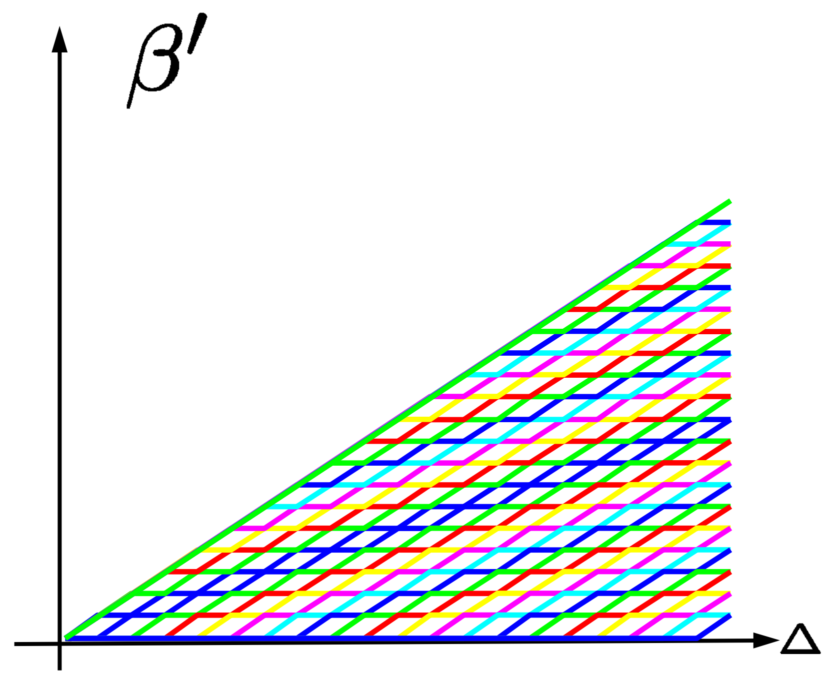


β'
Δ

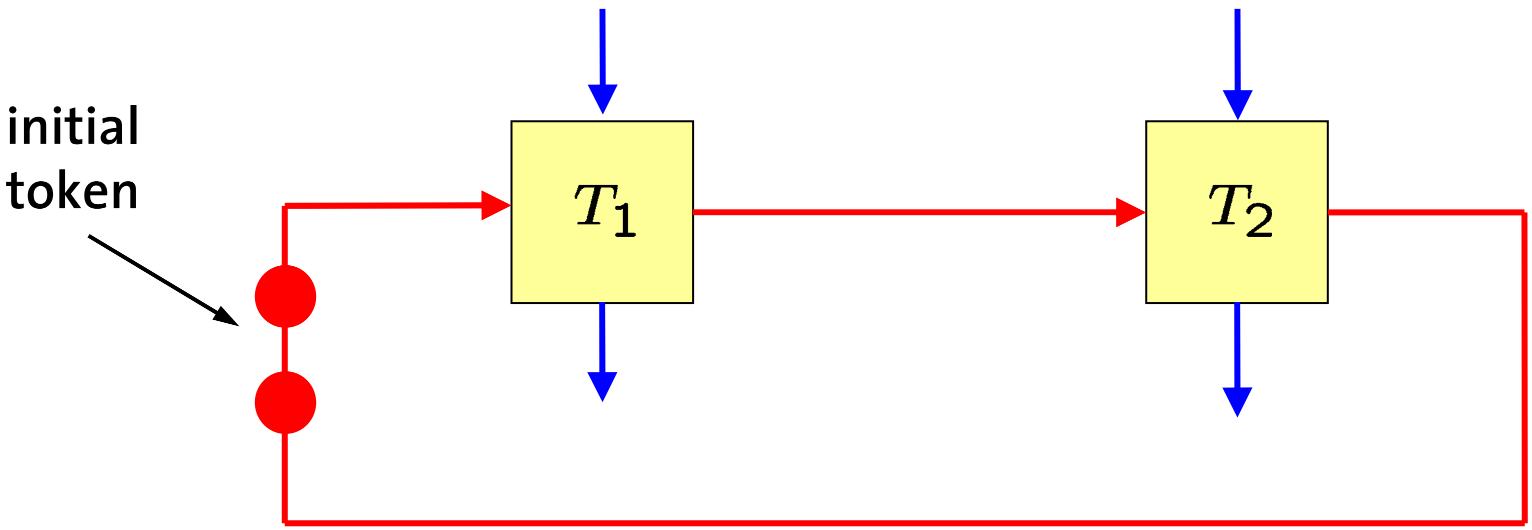
Open problem: Accuracy







Open Problem: Cyclic data-streams



Thank you!

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